Università degli Studi di L'Aquila - Dipartimento di Ingegneria e Scienze dell'Informazione e Matematica

Course catalogue

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Programme of Course "Geometria B"

- Code: DT0019
- Type of course unit: Compulsory (Bachelor Degree in Mathematics curriculum Generale)
- Level of course unit: Undergraduate Degrees
- Semester: 1

Number of ects credits: (Bachelor Degree in Mathematics) 12 (workload 300 hours)

Teachers: Alessandro Fedeli, Barbara Nelli

1	**Course objectives**	We foresee that the student will be able to deal with topology basic notions, that are necessary during the B.D. in mathematics. Moreover, after having learned a bunch of notions about curves and surfaces from the intrinsic and extrinsic viewpoint, the student will be able to solve problems about these topics.
2	**Course content and learning outcomes (dublin descriptors)**	Topics of the module include: • GENERAL TOPOLOGY: 1) Topological spaces, continuous functions and homeomorphisms. Metric spaces. 2) Basic constructions: subspaces, products and quotients. 3) Hausdorff spaces, separation properties and axioms of countability. 4) Compactness. Heine-Borel theorem. Tychonoff theorem. Sequential compactness. 5) Connectednes and path connectedness. 6) Brief introduction to topological manifolds, triangulations and classiffication of compact surfaces (Euler-Poincaré characteristic) • Differential geometry 1. Parametrized curves, regular curves and arc length. Jordan theorem (statement).Arcwise connection (definition). Frenet frame. Existence and uniqueness theorem for curves, given curvature and torsion. Local canonical form.2. Regular surfaces. Graphs, inverse images of regular values. Surfaces invariant by rotation. Differentiable maps between regular surfaces. Differential of a differentiable map. Tangent vector to a surface. The set of tangent vectors to a surface coincide with the image of R^2by the differential of a parametrization.3. Normal vector to a surfaceFirst fundamental form. Length of a curve on a surface, angles between two curves on a surface. Gauss map, differential of the Gauss map. Normal curvature, normal section. Second fundamental form. Principal curvatures, Olinde-Rodrigues theorem. Gauss and mean curvature. Points of a surface: elliptic, parabolic, hyperbolic, planar. 4. Gauss map in local coordinates. Dupin indicatrix. Asymptotic directions, Conjugate directions. Asymtpotic curves equations.Asymtotic curves for catenoids and helicoids. 5. Minimal surfaces: definition and characterization as critical points of the area functional. Isothermal parameters. Coordinates functions of a minimal surface are harmonic with respect to isothermal parameters. Equation of a minimal graph. Scherk's surface.6. Isometry and local isometry between surfaces. Two surfaces are locally isometric if they have parametrizations with equal coefficients of the first fundamental form.Conformal map and locally conformal map. Statement: two regular surfaces are always locally conformal. Christoffel symbols, Gauss equation and Codazzi-Mainardi equations. Egregium theorem.Fundamental theorem of local theory of surfaces (without proof).7. Tangent vector fields. Differentiability of a tangent vector field. Covariant derivative. Parallel vector fields.Vector fields along a curve. Parallel transport.Parallel transport is an isometry. Geodesic curvature. Algebraic value of the covariant derivative. Differential equations of the geodesics.8. Triangulation of a surface. Some notions about classification of compact surfaces. Genus of a surface.Gauss Bonnet theorem: local and global. Application of Gauss Bonnet theorem.Jacobi theorem.9. Differentiable vector field on a surface. Hopf-Poincare theorem and application.10. Exponential map on a surface and theorems.Geodesic coordinates and normal neighborhood. Minding theorem. Length of a geodesic circle. Computaton of the Gauss curvature in terms of the length of geodesic circles. Minimization properties of the geodesics. Rigidity of the sphere. Some generalization: Hopf and Alexandrov's theorem (only some notions) On successful completion of this module, the student should :

		• The student should have deep knowledge of the theory of curve and surfaces immersed in R3 and good knowledge of the basic notions of intrinsic geometry of surfaces. Moreover the student should acquire a sound knowledge of all basic notions and concepts of general topology. • The student should be able to solve problems about the theory of curves and surfaces immersed in R3 and some problem about intrinsic geometry of surfaces Moreover the student should be able to recognize when the acquired notions of general topology are necessary to the comprehension of other topics. • The student should be able to understand problems of curves and surfaces theory and topology and recognize the best method to solve them. • The student should be able to explain the statements and the proof s of the theorems about curves, surfaces and topology • The student should have acquired the ability of reading and understanding more advanced intrinsic theory of surfaces and topology.
3	**Course prerequisites**	first year courses of B.D. in mathematics
4	**Teaching methodsand language**	Theoretical course **Language**: Italian **Reference textbooks** • M. Abate, F. Tovena, ***Curve e Superfici***. Springer. • M. P. Do Carmo, ***Differential Geometry of Curves and Surfaces***. Prentice Hall. • V. Checcucci, A. Tognoli, E. Vesentini, ***Lezioni di Topologia Generale***. Feltrinelli.
5	**Assessment methods**	The exam is as follows: first a written exam and, if it is good, the student is allowed to participate to an oral exam.