Università degli Studi di L'Aquila - Dipartimento di Ingegneria e Scienze dell'Informazione e Matematica

Course catalogue

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Programme of Module "Mathematical Models for Collective Behaviour"

- Code: DT0013
- Type of course unit: Elective (Master Degree in Mathematical Engineering curriculum Comune)
- Level of course unit: Postgraduate Degrees
- Semester: 1

Number of ects credits: (Master Degree in Mathematics) 6 (workload 150 hours), (Master Degree in Mathematical Engineering) 6 (workload 150 hours)

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1	**Course objectives**	Aim of the course is to present some mathematical models currently used in the analysis of collective phenomena, such as vehicular and pedestrian traffic, and flocking phenomena. Emphasis will be given to the mathematical treatment of specific problems coming from real world applications.
2	**Course content and learning outcomes (dublin descriptors)**	Topics of the module include: • Macroscopic traffic models. LWR model, its derivation. Fundamental diagrams. The Riemann problem, examples. Second order models for traffic flow: Payne-Whitham model, description, drawbacks; Aw-Rascle model, shocks description, domains of invariance, instabilities near vacuum. • Theory: systems of conservation laws, strict hyperbolicity, Rankine-Hugoniot conditions; Lax admissibility condition. The Riemann problem for systems: the linear case; GNL and LD fields; rarefactions and contact discontinuities. BV functions, examples and properties. A compactness theorem. • Wave front tracking algorithm: approximate rarefactions, possible types of interactions. Bounds on number of waves and on total variation. Compactness of approximate solutions. The initial-boundary value problem on the half line: boundary Riemann problem, interactions with the boundary, control of the total variation by means of a Lyapunov-type functional. The Toll gate problem. • Networks, basic definitions, conservation of the flux. Examples. Distributions along the roads, maximization of the flux. Riemann problem on a junction composed by 2 incoming roads and 2 outgoing roads. The case of 2 incoming roads and 1 outgoing road: the "right of way" rule. Junction between one incoming and one outgoing road, different fluxes. • Pedestrian flow: normal and panic situation. Macroscopic models for evacuation, conservation of "mass", eikonal equation. The Hughes model for pedestrian flow. The eikonal equation: non uniqueness, viscosity solutions, relation with vanishing viscosity approximation. The Hughes model in one space dimension. Curve of turning points, Rankine-Hugoniot conditions. The case of constant initial density and of symmetric initial data; conservation of the left and right mass; an example with mass exchange across the turning point. Macroscopic models for pedestrian flow that include: knowledge of a preferred path, discomfort from walking along walls, tendency of avoiding high densities of pedestrian in a neighborhood (nonlocal term of convolution type), angle of vision, obstacle in the domain. Linearized stability around a smooth solution. • Introduction to the theory of flocking. Examples: Krause model for opinion dynamics, Cucker-Smale model, model for attraction-repulsion phenomena. The Cucker-Smale flocking model: basic properties, estimates on the kinetic energy. A "flocking theorem": proof by bootstrapping method (Ha and Tadmor). Some drawbacks of the model. Introduction to the kinetic limit for flocking: the N-particle distribution function, Liouville equation, marginal distribution, continuity equation. The formal mean-field limit: a Vlasov-type equation. On successful completion of this module, the student should : • Acquaint with basic mathematical models that describe collective phenomena, and with standard techniques in solving specific features • Demostrate skill in analyzing and interpret properly a mathematical model for

		collective behaviour • Understand and explain the meaning of other models using proper mathematical notation • Demonstrate capacity for reading and understanding other texts on related topics.
3	**Course prerequisites**	Basic notions of functional analysis. Classical solutions of partial differential equations of first order, method of characteristics.
4	**Teaching methodsand language**	Lectures, exercises **Language**: English **Reference textbooks** • M.D. Rosini, ***Macroscopic models for vehicular flows and crowd dynamics: theory and applications***. Springer. 2013. *http://link.springer.com/book/10.1007/978-3-319-00155-5/page/1* • M. Garavello, B. Piccoli, ***Traffic flow on networks. Conservation laws models***. AIMS Series on Applied Mathematics. 2006. *http://www.aimsciences.org/books/am/AMVol1.html*
5	**Assessment methods**	Written and oral examination