Università degli Studi di L'Aquila - Dipartimento di Ingegneria e Scienze dell'Informazione e Matematica

Course catalogue

Page compiled on 21/02/2020

Programme of Course "Dynamical systems and bifurcation theory"

- Code: I0459
- Type of course unit: Compulsory (Master Degree in Mathematical Engineering curriculum Comune)
- Level of course unit: Postgraduate Degrees
- Semester: 1

Number of ects credits: (Master Degree in Mathematical Engineering) 6 (workload 150 hours)

Teachers: Bruno Rubino (bruno.rubino@univaq.it)

1	**Course objectives**	To render the students able to solve practical problems of bifurcation analysis. To stimulate the emergence of an engineering critical sense in using algorithms of Applied Mathematics finalized to system design.
2	**Course content and learning outcomes (dublin descriptors)**	Topics of the module include: • LINEAR SYSTEMS OF DIFFERENTIAL EQUATIONS. Exponential of a linear operator, fundamental matrix. Stability for linear systems: stable, unstable and center subspaces. Sinks and sources. Classification of the equilibrium point for a planar linear system: saddles, nodes, foci and centers. Phase portrait in dimension two. Non homogeneous linear systems. • LOCAL THEORY FOR NON LINEAR SYSTEMS OF DIFFERENTIAL EQUATIONS. Invariant sets. Hyperbolic equilibrium points. Linearization. Stable manifold theorem. Center manifold theorem. TheHartman-Grobman theorem. Hyperbolic equilibrium points for planar systems: saddles, nodes, foci and centers. Stability and Liapunov function. Non hyperbolic equilibrium points for a planar system. Center manifold theory. Normal form theory. Gradient and Hamiltonian systems. • GLOBAL THEORY FOR NON LINEAR SYSTEMS OF DIFFERENTIAL EQUATIONS. Dynamical systems. Limits sets and attractors. Periodic orbits, limit cycles and separatrix cycles. Homoclinic and heteroclinic orbits. Compound separatrix cycles. Dulac theorem. Poincaré map for a cycle. Poincaré map for a focus. Planar case and Poincaré map. The Stable manifold theorem fo periodic orbits. Floquet theorem. The Center manifold theorem for periodic orbits. Liouvile theorem for the fundamental matrix. Characteristic exponents and multipliers for a period orbit. On successful completion of this module, the student should : • be able to understand the general concept of a dynamical system, and the significance of dynamical systems for modelling real world phenomena; • be able to analyze simple dynamical systems to find and classify regular behaviour; • be able to choose and use methods appropriate to describe, qualitatively and quantitatively, the solution sets of ordinary differential; • be able to analyse simple dynamical systems to find and classify regular behaviour, sketch phase portraits and determine stable and unstable manifolds; • be able to determine the existence and stability of periodic orbits in two dimensions using Dulac's criterion, Lyapunov functions, the Poincaré-Bendixson Theorem and Floquet theory; • be able to use the Centre Manifold Theorem to reduce the order of a system appropriately and bring the reduced system into normal form; • be able to identify and classify codimension one bifurcations of fixed points and sketch bifurcation diagrams; • be able to use bifurcation theory to qualitatively describe how dynamical systems taken from applications in science and technology depend on a parameter; • be familiar with some of the simpler bifurcation scenarios, and how they can be analyzed.
3	**Course prerequisites**	Linear Algebra, basic theory of Ordinary differential equations and basic of continuum mechanics.
4	**Teaching methodsand**	Lectures and exercises. The module is split into two parts: Dynamical Systems (Prof. Rubino, 4 ECTS) and Bifurcation Theory (Prof. Luongo, 2 ECTS)

	language	**Language**: English **Reference textbooks** • Lawrence Perko, ***Differential Equations and Dynamical Systems***. Springer. 2001.
5	**Assessment methods**	Written and oral.