Università degli Studi di L'Aquila - Dipartimento di Ingegneria e Scienze dell'Informazione e Matematica

Course catalogue

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Programme of Module " Partial differential equations"

- Code: DT0150
- Type of course unit: Compulsory (Bachelor Degree in Mathematics curriculum Generale)
- Level of course unit: Undergraduate Degrees
- Semester: 1

Number of ects credits: (Bachelor Degree in Mathematics) 6 (workload 150 hours)

Teachers: Corrado Lattanzio

1	**Course objectives**	EQUATIONS OF MATHEMATICAL PHYSICS Students will know basic of properties (existence, uniqueness, etc.) and techniques (characteristics, separation of variables, Fourier methods, Green's functions, similarity solutions, etc.) to solve basic PDEs (conservation laws, heat, Laplace, wave equations).
2	**Course content and learning outcomes (dublin descriptors)**	Topics of the module include: • Integral curves and surfaces of vector fields. First order partial differential equations. Linear and quasi linear partial differential equations (PDEs) of first order. Method of characteristics. The initial value problem: existence and uniqueness. Development of shocks. • The Cauchy-Kovalevsky theorem. Linear partial differential operators and their characteristic curves and surfaces. Methods for finding characteristic curves and surfaces. The initial value problem for linear first order equations in two independent variables. Holmgren's uniqueness theorem. Canonical form of first order equations. Classification and canonical forms of second order equations in two independent variables. Second order equations in two or more independent variables. The principle of superposition. • The divergence theorem and the Green's identities. Equations of Mathematical Physics. • LAPLACE'S EQUATION AND HARMONIC FUNCTIONS Elementary harmonic functions. Separation of variables. Inversion with respect to circles and spheres. Boundary value problems associated with Laplace's equation. Representation theorem. Mean value property. Maximum principle. Harnack’s inequality and Liouville’s theorem. Well-posedness of the Dirichlet problem. Solution of the Dirichlet problem for the unit disc. Fourier series and Poisson's integral. Analytic functions of a complex variable and Laplace's equation in two dimensions. The Neumann problem. • GREEN'S FUNCTIONS. Solution to the Dirichlet problem for a ball in three dimensions. Further properties of harmonic functions. The Dirichlet problem in unbounded domains. Method of electrostatic images. • THE WAVE EQUATION. Cauchy problem. Energy method and uniqueness. Domain of dependence and range of influence. Conservation of energy. One-dimensional wave equation. D’Alembert formula. Characteristic parallelogram. Non homogeneous equation and Duhamel’s method. Multi-dimensional wave equation. Well posed problems. Fundamental solution (n=3) and strong Huygens’ principle. Kirchhoff formula. Method of descent. Poisson?s formula (n=2). Wave propagation in regions with boundaries. Uniqueness of solution of the initial-boundary value problem. Separation of variables. Reflection of waves. • THE HEAT EQUATION. Heat conduction in a finite rod. Maximum principle and applications. Solution of the initial-boundary value problem for the one dimensional heat equation. Method of separation of variables. The initial value problem for the one dimensional heat equation. Fundamental solution. Non homogeneous case and Duhamel’s method. Heat conduction in more than one space dimension. On successful completion of this module, the student should : • have advanced knowledge of classical theory for first and second order PDEs of Mathematical Physics; • have basic notions of fluid mechanics from an engineering point of view; • be able to understand when an elementary problem from Mathematical Physics is

		well posed; • be able to solve classical problems coming from Mathematical Physics such as initial, boundary, initial-boundary value problems; • be able to learn autonomously additional results for PDEs of Mathematical Physics.
3	**Course prerequisites**	The student must know the basic notions of mathematical analysis, including Fourier series and ordinary differential equations and the basic notions of continuum mechanics.
4	**Teaching methodsand language**	Lectures and exercise sessions. **Language**: English **Reference textbooks** • S. Salsa, G. Verzini, ***Equazioni a derivate parziali: complementi ed esercizi***. Springer-Verlag Italia. 2005. • E. C. Zachmanoglou and Dale W. Thoe, ***Introduction to Partial Differential Equations with Applications***. Dover Publications, Inc.. 1986. • L.C. Evans, ***Partial Differential Equations***. American Mathematical Society. 2010. • S. Salsa, ***Partial Differential Equations in Actions: from Modelling to Theory***. Springer-Verlag Italia. 2008. • W. A. Strauss, ***Partial Differential Equations, Student Solutions Manual: An Introduction***. John Wiley & Sons, LTD. 2008. • W. A. Strauss, ***Partial Differential Equations: an introduction***. John Wiley & Sons, LTD. 2007.
5	**Assessment methods**	Written and oral.