Summary of lectures [1]	
Introduction to the course	Data analytics, decision models to limit data misuse. Data types: raw data, dispersions, distributions, time series; types of analysis: descriptive, predictive, prescriptive. Examples of descriptive parameters: mean, median, mode, variance, etc.
Distributions, caveats	How to construct a histogram: Excel frequency function. Density and cumulative distributions. Caveats: statistics vs. probability. Queuing systems, the Waiting Time Paradox. Population vs. sample variance, Bessel's correction. Numerical accuracy and chaotic systems.
Time series	Time series (signals): Fourier transform and anti-transform, limited band signals, Sample Theorem (enunciate).
Time series	Examples of descriptive analytics of time series: cross- and self-correlation. A numerical case: computing self-correlation of a finite/discrete time series via Toeplitz matrix.
Location models	Clustering a dispersion of records: a *p*-median location model with one decision maker. Single-objective vs. Multi-objective problems. Pareto-optimality. Multiple decision makers with non-converging objectives. Location and pricing problem as a Stackelberg game.
Location models	Formulation of *p*-median, *p*-centre, *p*-(median+centre) problems as integer linear programs. Numerical example, size of solution space.
Optimization problems and models	Definitions: solution feasibility and optimality, problem consistency and unboundedness. Convexity of feasible set. Linear optimization (or LP), geometric vs. analytical description of the feasible set, convex polyhedra. Consistency check: the projection principle.
Optmization problems and models	An example of linear optimization: drawing a convex polyhedron.
Convex polyhedra	Numerical examples.
Convex polyhedra	Affine and linear subspaces, hyperplanes, half-spaces. Special polyhedra: empty set, hyperplane, polyhedral cone. Linear, affine, conic and convex combination. Convex, conic and affine hull. Projection of a polyhedron. Fourier-Veronese Theorem (enunciate only). Examples.
Fourier-Motzkin Method	More numerical examples. Worst case analysis of algorithm complexity.
Duality Theory	From Fourier-Veronese Theorem to the Theorem of the Alternative. Dual (and primal) LP. Weak Duality Theorem. Corollaries. Strong Duality Theorem.
Duality Theory	How to construct the dual of an LP. Numerical examples.
Elements of graph theory	Graphs, directed and undirected. Degree. Regular graphs. Paths, cycles, trees. Connectivity. Handshaking Theorem and Euler Theorem. Eulerian and Hamiltonian graphs.
Duality Theory	Examples of dual problems. Rules for constructing the dual problem. Interpretation of the dual problem. Production optimization and shadow prices. Portfolio optimization and zero-sum games.
Solving LPs	Solving LPs by Fourier-Motzkin Projection Method. Examples: a generic LP, a portfolio optimization problem. Discussion on complexity.
Zero-sum games	Portfolio optimization as a zero-sum game. Dual analysis.
Combinatorial problems	The Transversal problem. 01 LP formulation.
Elements of graph theory	Making a graph Eulerian. An application. Connectivity, forests, trees. Ramsey Theorem. Bipartite graphs and their characterization.
Combinatorial problems	Basic optimization problems on graphs: stable set, clique, transversal, matching, edge-cover, dominating set, coloring. Examples, applications and 01 LP formulations.
Classification in data mining	An example of bad classification. A methodology for building sensible classes (via optimal graph coloring).
Combinatorial problems	Stability, transversal, matching and edge-cover numbers. Primal-dual relations. Edge and clique formulation of the stable set problem. Clique-cover number. Clique and chromatic numbers.

[1] Lectures in black are intended for both courses. Additional lectures in blue are intended for *Optimization Models and Algorithms* only.

Regression	Regression curves in the plane: linear and polynomial regression; generalization. Regression curves in *n*-dimensional spaces.
Implicit enumeration methods	The curse of dimensionality. Introduction to implicit enumeration: branch-and-bound. Combinatorial optimization and 01 LP, branching by dichotomy. Use of bounds. Numerical example: stable set problem.
Interpolation of efficiency curve	An LP model to interprete the region of non-dominated points. Numerical example in the plane.
Separation	Finding a separating hyperplane: an LP model. An ILP model for separation with a minimum number of outliers.
Matroids and the Greedy Algorithm	Introduction to the Greedy Algorithm. Packing and covering problems. Other combinatorial problems. Two encodings of the Greedy Algorithm. Why does the Greedy Algorithm fail in general?
Implicit enumeration	Numerical example: solving a 01 knapsack problem by branch-and-bound.
Implicit enumeration methods	Branch-and-bound methods. General features: problem partition and use of bounds. Bounds by linear relaxation. Combinatorial bounds, example: TSP and spanning tree bound.
Convex hull of a discrete set	Numerical exercise: compute the implicit form of *conv*(*S*) by Fourier-Motzkin projection method.
Overlap	Ways to compute the overlap of two non-separable sets. Use of convex hull. Use of separation hyperplanes.
Separation in 01 LP	Separation of an infeasible solution. Example: TSP. Separation by subtour elimination constraints as a max-flow min-cut problem.
Matroids	Rado's Theorem.
Efficient clustering	Motivation. Tree-clustering. Hierarchical tree-clustering. Applications.
Matroids	Rado’s Theorem (enunciate) Consequences of Rado's Theorem, examples. Graphic matroid.
Dynamic programming	Precedence relations and DAGs. Topological ranking. Shortest, longest, safest paths. Levenshtein distance.
Protein design via 01LP	Basics of protein structure: amino-acids and codons. Codon optimization and motif engineering. Dynamic programming vs. 01 LP. Numerical evidence.
VLSI design via 01LP: a tutorial	Boolean functions and PLA. Reducing device area: row compatibility. A combinatorial optimization model.
Matroids and the Greedy algorithm	More matroids: partition matroid, assignment, linear matroid. Representable matroids: graphical as a linear matroid, basic solution of a network flow problem.
Bipartite matching	Matchings and augmenting paths. A combinatorial algorithm for unweighted bipartite matching and its computational complexity. Shortest augmenting paths: extension to the weighted case, complexity.
Matroids and the Greedy algorithm	Examples of application: optimal bidding in constructing a network infrastructure. Two matroid intersection, bipartite matching.
Totally unimodular matrices	Unimodular and totally unimodular matrices. Examples and motivation. Sufficient conditions. Consequences on matching, stable set, transversal and edge-cover problems on bipartite graphs. Non-bipartite matching and odd cycles inequalities. Consecutive one property and stable set/clique cover on interval graphs.