DECISION MODELS Final test of Feb 19th, 2019

Given name: |__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|

Family name: |__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|

Id. number: |__|__|__|__|__|__|

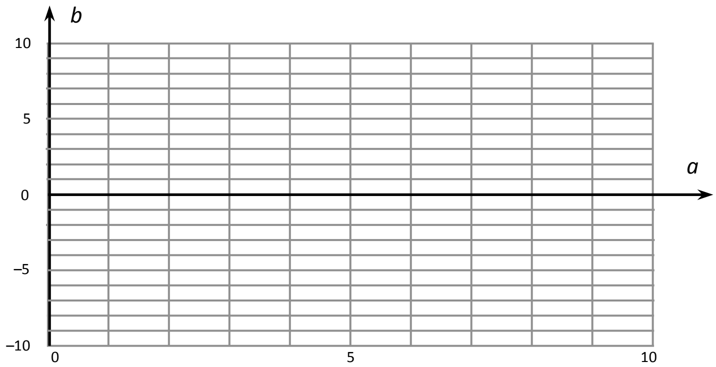
1. The 01 LP formulation

$$\min \sum_{ij \in I\text{x}I} d_{ij}\, x_{ij}$$

$$\begin{array}{llll} x_{ij} - x_i - x_j & \geq & -1 & \text{for all } ij \in I\text{x}I \\ x_{ij} + x_i + x_j & \geq & 1 & \text{for all } ij \in I\text{x}I \\ x_i, x_{ij} \in \{0, 1\} & & & \text{for all } i \in I \text{ and for all } ij \in I\text{x}I \end{array}$$

refers to a square non-negative matrix $\mathbf{D} = \{d_{ij}\}$, indexed on a set I, whose entries represent evaluations of dissimilarities. What is the problem it describes?

2. Consider the regression line r of $(x_1, y_1) = (1, 5)$, $(x_2, y_2) = (2, 8)$, $(x_3, y_3) = (3, 7)$, where the distance d of r from the set is defined as the largest difference (in absolute value) between y_k and the value $ax_k + b$ that r has in x_k, for $k = 1, 2, 3$. Is this distance ≥ 1? Find the answer by writing the system of inequalities that defines r, and drawing the corresponding polyhedron in the grid aside.



3. For each pair i, j of points of a given set N, a parameter $s_{ij} \geq 0$ is known: the largest the s_{ij}, the more i and j feel similar to each other. We define the global similarity of a set K as

$$s(K) = \sum_{i \in K} \sum_{j \in K} s_{ij}$$

We wish to select K with $|K| = k$ and such that $s(K)$ is maximum. To select K, introduce binary variables x_i for each point i. To write a linear expression of $s(K)$, introduce also variables x_{ij} for each pair i, j. Write then suitable linear inequalities that constrain the x_i to the x_{ij} so that $x_{ij} = 1$ if and only if $x_i = x_j = 1$. Complete the formulation with the objective function and all the necessary constraints.