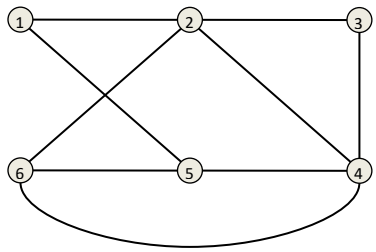
DECISION MODELS **Intermediate test, Nov 7th 2019**

Given name: |__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|

Family name: |__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|

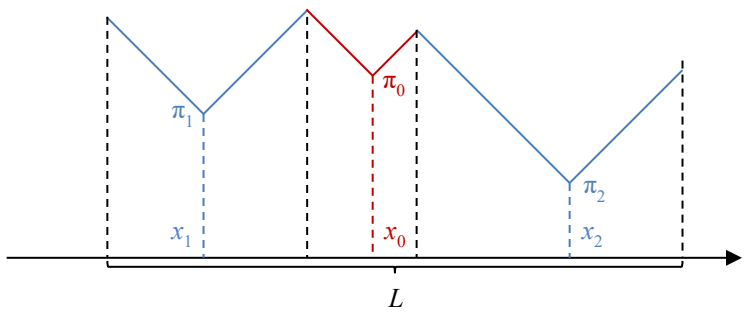
Id. number: |__|__|__|__|__|__|

1. **Stable set**. Using decision variables x_u associated with the vertices of the graph $G = (V, E)$ in the figure, formulate the problem of finding the largest stable set of G. Remove then the integrality clauses, write the dual, reintroduce the integrality clauses in the resulting problem and derive an interpretation of the problem D so obtained. Then use duality theorems to derive a relation between the size $\alpha(G)$ of the largest stable set of G and the optimal value $\rho(G)$ of the objective function of D.



2. **Projection**. Take the stable set formulation above, relax integrality clauses and think of solving it by Fourier-Motzkin method by first projecting out variable x_6. Reintroduce at this point integrality and describe the graph optimization problem P that you obtained.

3. **Spatial competition**. Two shops lie on a road (represented as an interval of length L, see figure) in positions x_1, x_2, and offer the same service to customers at prices π_1, π_2. N customers live along the road, and the probability that a local customer lives in a point x equals N/L independently on x: said in other words, the road offers N/L customers per unit length. To be served by shop k ($k = 1, 2$), a customer in x bears a cost equal to $\pi_k + |x - x_k|$.



Suppose we open a shop in $x_0 \in [x_1, x_2]$ and offer the same service at price π_0. Describe an optimization model aiming at choosing the value of π_0 that maximizes our expected revenue subject to optimal customers' decision. Then:

   a. Compute the optimal price for $\pi_1 = 120$, $\pi_2 = 100$, $L = 200$, $x_1 = 30$, $x_2 = 170$, $x_0 = 50$.
   b. Extend the model to a location-pricing problem, that is, to the case in which also x_0 is a decision variable.

4. **Regression**. Write a linear optimization model to interpret the trend $(x, f(x))$ of a dispersion $\{(x_1, y_1), \ldots, (x_n, y_n)\} \subseteq \mathrm{IR}^2$ with a sigmoidal curve

$$f(x) = (1 + e^{-x})^{-a} \qquad (a > 0)$$

Assume the distance between the curve and a point (x_k, y_k) given by $|y_k - f(x_k)|$.