

DIEGO ALBERICI

born on December 5th, 1988 in Bologna (Italy)

working address: DISIM – Università dell’Aquila
via Vetoio, edificio “Coppito1”
67100 L’Aquila
Italia

e-mail: diego.alberici@univaq.it

POSITIONS

- 1/12/2021 – now Ricercatore di tipo B (tenure track position)
DISIM, **Università dell’Aquila**
- 1/4/2020 – 30/11/2021 Collaborateur scientifique (postdoc researcher)
Laboratoire de théorie des communications,
École Polytechnique Fédérale de Lausanne
- 1/4/2018 – 31/3/2020 Assegnista di ricerca (postdoc researcher)
Dipartimento di Matematica, **Università di Bologna**
- 1/3/2017 – 28/2/2018 Assegnista di ricerca (postdoc researcher)
Dipartimento di Matematica, **Università di Bologna**
- 1/1/2016 – 31/12/2016 Assegnista di ricerca (postdoc researcher)
Dipartimento di Matematica, **Università di Bologna**

ABILITAZIONE SCIENTIFICA NAZIONALE

- 9/9/2019 Italian scientific qualification for associate professor in Mathematical
Physics (II fascia s.c. 01/A4)

EDUCATION

- 1/2013 - 29/4/2016 **Ph.D. in Mathematics**
Università di Bologna
thesis "*Statistical Mechanics of hard-core particles with attractive interactions*",
supervisor Prof. Pierluigi Contucci, examination board: Prof. André Martinez,
Prof. Alessandro Giuliani, Prof. Elliott Lieb
- 8/2014 - 12/2014 **visiting student research collaborator at Princeton University**
invited by Prof. Elliot Lieb
- 9/2010 - 28/9/2012 **Master in Mathematics**
Università di Bologna
110/110 with honors

thesis "*Statistical Mechanics of monomer-dimer models on complete and Erdős-Rényi graphs*", supervisor Prof. Pierluigi Contucci

9/2010 - 7/2011 **Erasmus student at Université Paris VII Denis Diderot**

9/2007 - 1/10/2010 **Bachelor in Mathematics**
Università di Bologna
110/110 with honors
thesis "*Il limite termodinamico nei sistemi di spin diluiti*",
supervisor Prof. Pierluigi Contucci

SCIENTIFIC PUBLICATIONS and PREPRINTS

[17] D. A., N. Macris, E. Mingione, "*The stationary non-equilibrium measure for a two-temperatures dynamics with two different time scales*", **preprint**

Abstract. A general interacting system composed by two subsystems in two different temperatures baths is considered. A two time scales stochastic dynamics is introduced for the two parts of the system and the corresponding Fokker-Planck equation is rigorously solved in a suitable limit of long time and very different time scales. The resulting stationary non-equilibrium measure describes interacting systems under general hypothesis on the confining potential. Examples of interest include soft-spin glasses and possible applications to Machine Learning.

[16] D. A., F. Camilli, P. Contucci, E. Mingione, "*A Statistical Physics approach to a multi-channel Wigner spiked model*", **Europhysics Letters** 136, 48001 (2022)

Abstract. In this letter we present a finite temperature approach to a high-dimensional inference problem, the Wigner spiked model, with group-dependent signal-to-noise ratios. For two classes of convex and non-convex network architectures the error in the reconstruction is described in terms of the solution of a mean-field spin-glass on the Nishimori line. In the cases studied the order parameters do not fluctuate and are the solution of finite dimensional variational problems. The deep architecture is optimized in order to confine the high temperature phase where reconstruction fails.

[15] D. A., F. Camilli, P. Contucci, E. Mingione, "*The solution of the deep Boltzmann machine on the Nishimori line*", **Communications in Mathematical Physics** 387, 1191-1214 (2021)

Abstract. The deep Boltzmann machine on the Nishimori line with a finite number of layers is exactly solved by a theorem that expresses its pressure through a finite dimensional variational problem of min-max type. In the absence of magnetic fields the order parameter is shown to exhibit a phase transition.

[14] D. A., F. Camilli, P. Contucci, E. Mingione, "*The multi-species mean-field spin-glass in the Nishimori line*", **Journal of Statistical Physics** 182, 156-176 (2021)

Abstract. In this paper we study a multi-species disordered model in the Nishimori line. The typical properties of this line, a set of identities and inequalities, allow us to prove the replica symmetry, i.e., the concentration of the order parameter. When the interaction structure is elliptic we rigorously compute the exact solution of the model in terms of a finite-dimensional variational principle.

[13] D. A., P. Contucci, E. Mingione, "*Deep Boltzmann machines: rigorous results at arbitrary depths*", **Annales Henri Poincaré**, 22, 2619-2642 (2021)

Abstract. A class of deep Boltzmann machines is considered in the simplified framework of a quenched system with Gaussian noise and independent entries. The quenched pressure of a K-layers spin glass model is studied allowing interactions only among consecutive layers. A lower bound for the pressure is found in terms of a convex combination of K Sherrington-Kirkpatrick models and used to study the annealed and replica symmetric regimes of the system. A map with a one dimensional monomer-dimer system is identified and used to rigorously control the annealed region at arbitrary depth K with the methods introduced by Heilmann and Lieb. The compression of this high noise region displays a remarkable phenomenon of localization of the processing layers. Furthermore a replica symmetric lower bound for the limiting quenched pressure of the model is obtained in a suitable region of the parameters and the replica symmetric pressure is proved to have a unique stationary point.

[12] D. A., A. Barra, P. Contucci, E. Mingione, "*Annealing and replica-symmetry in deep Boltzmann machines*", **Journal of Statistical Physics** 180, 665-677 (2020)

Abstract. In this paper we study the properties of the quenched pressure of a multi-layer spin-glass model (a deep Boltzmann Machine in artificial intelligence jargon) whose pairwise interactions are allowed between spins lying in adjacent layers and not inside the same layer nor among layers at distance larger than one. We prove a theorem that bounds the quenched pressure of such a K-layer machine in terms of K Sherrington-Kirkpatrick spin glasses and use it to investigate the annealed region. The replica-symmetric approximation of the quenched pressure is identified and its relation to the annealed one is considered. The paper also presents some observation on the model's architectural structure related to machine learning: escaping the annealed region is mandatory for a meaningful training and by squeezing such region we obtain thermodynamic constraints on the form factors. Remarkably, the optimal escape is achieved by requiring the last layer to scale sub-linearly in the network size.

[11] D. A., P. Contucci, E. Mingione, "*Mean field monomer dimer models. A review*", **Sojourns in Probability and Statistical Physics I**, V. Sidoravicious editor, 39-62 (2019)

Abstract. A collection of rigorous results for a class of mean-field monomer-dimer models is presented. It includes a Gaussian representation for the partition function that is shown to considerably simplify the proofs. The solutions of the quenched diluted case and the random monomer case are explained. The presence of the attractive component of the Van der Waals potential is considered and phase transition analyzed. In particular the breakdown of the central limit theorem is illustrated at the critical point where a non Gaussian, quartic exponential distribution is found for the number of monomers centered and rescaled with the volume to the power $3/4$.

[10] D. A., P. Contucci, R. Luzi, C. Vernia, "*Finite-size corrections for the attractive mean-field monomer-dimer model*", **Journal of Physics A: Mathematical and Theoretical** 52, 105001-105013 (2019)

Abstract. The finite volume correction for a mean-field monomer-dimer system with an attractive interaction are computed for the pressure density, the monomer density and the susceptibility. The results are obtained by introducing a two dimensional integral representation for the partition function decoupling both the hard-core interaction and the attractive one. The next-to-leading terms for each of the mentioned quantities are explicitly derived as well as the value of their sign that is related to their monotonic convergence in the thermodynamic limit.

[9] D. A., E. Mingione, "*Two populations mean-field monomer-dimer model*", **Journal of Statistical Physics** 117, 96-105 (2018)

Abstract. A two populations mean-field monomer–dimer model including both hard-core and attractive interactions between dimers is considered. The pressure density in the thermodynamic limit is proved to satisfy a variational principle. A detailed analysis is made in the limit of one population much smaller than the other and a ferromagnetic mean-field phase transition is found.

[8] D. A., P. Contucci, E. Mingione, M. Molari, "*Aggregation models on hypergraphs*", **Annals of Physics** 376, 412-424 (2017)

Abstract. Following a newly introduced approach by Rasetti and Merelli we investigate the possibility to extract topological information about the space where interacting systems are modeled. From the statistical datum of their observable quantities, like the correlation functions, we show how to reconstruct the activities of their constitutive parts which embed the topological information. The procedure is implemented on a class of polymer models on hypergraphs with hard-core interactions. We show that the model fulfills a set of iterative relations for the partition function that generalize those introduced by Heilmann and Lieb for the monomer–dimer case. After translating those relations into structural identities for the correlation functions we use them to test the precision and the robustness of the inverse problem. Finally the possible presence of a further interaction of peer-to-peer type is considered and a criterion to discover it is identified.

[7] D. A., P. Contucci, M. Fedele, E. Mingione, "*Limit theorems for monomer-dimer mean-field models with attractive potential*", **Communications in Mathematical Physics** 346, 781-799 (2016)

Abstract. The number of monomers in a monomer–dimer mean-field model with an attractive potential fluctuates according to the central limit theorem when the parameters are outside the critical curve. At the critical point the model belongs to the same universality class of the mean-field ferromagnet. Along the critical curve the monomer and dimer phases coexist.

[6] D.A., P. Contucci, E. Mingione, "*Non-Gaussian fluctuations in monomer-dimer models*", **Europhysics Letters** 114, 10006 (2016)

Abstract. A hard-core monomer-dimer mean-field model is considered with the addition of an attraction potential between similar particles. We find that in the curve where the monomer and dimer phases coexist, the equilibrium state, due to the lack of gauge symmetry, turns out to be a superposition with unequal weights. We show, moreover, that at the critical point the number of monomers has non-Gaussian, quartic exponential, fluctuations.

[5] D. A., "*A cluster expansion approach to the Heilmann-Lieb liquid crystal model*", **Journal of Statistical Physics** 162, 761-791 (2016)

Abstract. A monomer-dimer model with a short-range attractive interaction favoring collinear dimers is considered on the lattice \mathbb{Z}^2 . Although our choice of the chemical potentials results in more horizontal than vertical dimers, the horizontal dimers have no long-range translational order, in agreement with the Heilmann–Lieb conjecture (Heilmann and Lieb in *J Stat Phys* 20(6):679–693, 1979).

[4] D. A., P. Contucci, E. Mingione, "*A mean-field monomer-dimer model with randomness*", **Journal of Statistical Physics** 160, 1721-1732 (2015)

Abstract. Independent random monomer activities are considered on a mean-field monomer–dimer model. Under very general conditions on the randomness the model is shown to have a self-

averaging pressure density that obeys an exactly solvable variational principle. The dimer density is exactly computed in the thermodynamic limit and shown to be a smooth function.

[3] D. A., P. Contucci, E. Mingione, "*The exact solution of a mean-field monomer-dimer model with attractive potential*", **Europhysics Letters** 106, 10001-10005 (2014)

Abstract. A monomer-dimer model with an attractive interaction that favors a phase separation between monomers and dimers is exactly solved in the mean-field case. With the identification of a suitable variational principle the free energy is computed in the large volume limit using the Heilmann-Lieb pure hard-core ansatz. The monomer density, that turns out to be the order parameter of the model, is shown to have a first-order phase transition along a coexistence curve.

[2] D. A., P. Contucci, E. Mingione, "*A mean-field monomer-dimer model with attractive interaction. Exact solution and rigorous results*", **Journal of Mathematical Physics** 55, 063301 (2014)

Abstract. A mean-field monomer-dimer model which includes an attractive interaction among both monomers and dimers is introduced and its exact solution rigorously derived. The Heilmann-Lieb method for the pure hard-core interacting case is used to compute upper and lower bounds for the pressure. The bounds are shown to coincide in the thermodynamic limit for a suitable choice of the monomer density m . The computation of the monomer density is achieved by solving a consistency equation in the phase space (h, J) , where h tunes the monomer potential and J the attractive potential. The critical point and exponents are computed and show that the model is in the mean-field ferromagnetic universality class.

[1] D. A., P. Contucci, "*Solution of the monomer-dimer model on locally tree-like graphs. Rigorous results*", **Communications in Mathematical Physics** 331, 975-1003 (2014)

Abstract. We consider the monomer–dimer model on sequences of random graphs locally convergent to trees. We prove that the monomer density converges almost surely, in the thermodynamic limit, to an analytic function of the monomer activity. We characterize this limit as the expectation of the solution of a fixed point distributional equation and we give an explicit expression of the limiting pressure per particle.

INVITED TALKS

- "*Convergence of a multi-bath dynamics to a (non-equilibrium) stationary measure and connections with spin glass theory*"
conference: **G.N.F.M. scientific meeting** (italian group of Mathematical Physics)
Montecatini, 05/05/2022
- "*A multi-bath (non equilibrium) stationary measure: possible connections with Spin Glass theory, Statistical Inference, and Functional Analysis*"
Università dell'Aquila, 22/12/2021
- "*Random deep Boltzmann machine: analytic results about the replica symmetric phase*"
conference: **"I Convegno della Società Italiana di Fisica Statistica - XXV Convegno Nazionale di Fisica Statistica e dei Sistemi Complessi"**
Università di Parma, 23/06/2021
- "*Multi-layer SK model: the annealed and replica symmetric regimes*"
series of seminars: "Seminars in Probability Theory and Statistics"

University of Basel, 18/11/2020

- “*Statistical Mechanics & Complex Zeros: it's a match!*”
series of seminars: “Ba.D. seminars” organized by Ph.D. students and postdocs
Università di Bologna, 27/11/2019
- “*Random deep Boltzmann machines: the annealed phase*”
conference: “**Complex Simplex: Topological and Network Data Science**”
Politecnico di Torino, 15/10/2019
- “*Monomer-dimer models in random environment*”
conference: “**Mathematical Physics of Disordered Systems**”
Università di Bologna, 25/06/2019
- “*Monomer-dimer models on random graphs*”
Ecole Polytechnique Fédérale de Lausanne, Lausanne, 28/05/2019
- “*Monomer-dimer models on random graphs*”
International Center for Theoretical Physics, Trieste, 14/05/2019
- “*Monotonicity and finite-size corrections in mean-field monomer-dimer models*”
conference: **G.N.F.M. scientific meeting** (italian group of Mathematical Physics)
Montecatini, 4/10/2018
- “*Liquid crystals: the Heilmann-Lieb conjecture*”
N.Y.U. Abu Dhabi, 14/12/2017
- “*Complex zeros can melt ice: a theorem by Lee-Yang on the edge of Statistical Physics, Algebra and Complex Analysis*”
series of seminars: “Ba.D. seminars” organized by Ph.D. students and postdocs
Università di Bologna, 6/12/2016
- “*Liquid crystals: a cluster expansion approach to the Heilmann-Lieb conjecture*”
conference: “**XXI convegno nazionale di Fisica Statistica e Sistemi Complessi**”
Università di Parma, 29/06/2016
- “*Monomer-dimer models on a class of random graphs*”
conference: **G.N.F.M. scientific meeting** (italian group of Mathematical Physics)
Montecatini, 24/10/2015
- “*Monomer-dimer models on a class of random graphs*”
Università di Ferrara, 24/09/2015
- “*Monomer-dimer models on a class of random graphs*”
conference: “**Disordered systems, random spatial processes and some applications**”
trimestre dell’Institut Henri Poincaré, Paris, 19/02/2015
- “*Modelli di monomero-dimero su una classe di grafi random*”
Università di Modena, 10/06/2013

WORKSHOPS ORGANIZATION

I was in the organizing committee of the following conferences:

- **“Mathematical Methods and Models in Machine Learning”**
Università di Bologna, 27-28 April 2020
<https://eventi.unibo.it/m4l2020/>
- **“Mathematical Physics of disordered systems”**
Università di Bologna, 24-25 June 2019
<https://eventi.unibo.it/disordered-systems-bologna2019/>

TEACHING ACTIVITY

- **professor of Meccanica Razionale**
Bachelor in Mathematics, Università dell’Aquila, 2021/2022
60 hours
- lessons for **Quantum Information Theory and Computation**
Doctoral School, EPFL, 2021/2022
4 hours
- **teaching tutor of Meccanica Statistica dei sistemi complessi**
Master in Mathematics, Università di Bologna, 2018/2019
15 hours
- **adjunct professor of Calculus and Linear Algebra**
Bachelor in Economics and Finance, Università di Bologna, 2017/2018
45 hours
- **teaching tutor of Meccanica Statistica dei sistemi complessi**
Master in Mathematics, Università di Bologna, 2017/2018
15 hours
- **teaching tutor of Meccanica Statistica dei sistemi complessi**
Master in Mathematics, Università di Bologna, 2016/2017
15 hours
- **teaching tutor of Fisica Matematica 1**
Bachelor in Mathematics, Università di Bologna, 2016/2017
30 hours
- **teaching tutor of Calculus and Linear Algebra**
Bachelor in Economics and Finance, Università di Bologna, 2015/2016
50 hours
- seminars for **Meccanica Statistica dei sistemi complessi** about monomer-dimer models,
Università di Bologna, 2014/2015
4 hours
- seminars for **Meccanica Statistica dei sistemi complessi** about the central limit theorem,
Università di Bologna, 2013/2014

4 hours

- seminars for **Matematica per le applicazioni socio-economiche** about the Ising model on Erdős-Rényi random graphs, Università di Bologna, 2012/2013
4 hours
- **teaching tutor** of **Calcolo delle Probabilità e Statistica**
Bachelor in Informatics, Università di Bologna, 2011/2012
80 hours
- help with **oral exams** of **Fisica Matematica 1**
Bachelor in Mathematics, Università di Bologna, 2019/2020
- help with **oral exams** of **Fisica Matematica 1**
Bachelor in Mathematics, Università di Bologna, 2018/2019
- help with **oral exams** of **Fisica Matematica 1**
Bachelor in Mathematics, Università di Bologna, 2017/2018
- **tutor** of **“Piano Lauree Scientifiche”** (laboratory project for high school students aimed at spreading scientific degrees)
Department of Mathematics, Università di Bologna, 2015/2016
- **tutor** for the students of the **Bachelor in Mathematics**, tutor of **“Piano Lauree Scientifiche”**, assistance for graduation sessions
Department of Mathematics, Università di Bologna, 2011/2012
120 hours

GRANTS and AWARDS

- member of an **AlmaIdea project for junior researchers** - “Transizioni di fase nei modelli disordinati”, Università di Bologna 2018-2020 (20.000 euros)
- **coordinator** of a **Progetto Giovani GNFM INdAM** (grant for young researchers set up by the national group of Mathematics Physics) - “Approccio meccanico-statistico ad una teoria di campo dei dati”, 2017-2018 (2.500 euros)
- **mention** at **Claudio Bonivento award for the best scientific PhD thesis**, Università di Bologna 2017
- member of **PRIN 2015K7KK8L_003** (italian public grant) - “Meccanica Statistica e complessità” 2017-2020
- member of a **Progetto Giovani GNFM INdAM** (grant for young researchers set up by the national group of Mathematics Physics) - “Misure di Gibbs nei modelli di monomero dimer con interazione” 2015-2016 (2.000 euros)
- member of a **Progetto Giovani GNFM INdAM** (grant for young researchers set up by the national group of Mathematics Physics) - “Investigations on the mean-field monomer-dimer model with attractive interaction”, GNFM INdAM 2014-2015

- winner of a **Marco Polo grant to visit Princeton University**, set up by Università di Bologna 2014
- member of **PRIN 2010HXAW77_010** (italian public grant) - “Meccanica Statistica dei sistemi disordinati e complessi” 2013-2016
- winner of a **Ph.D. scholarship**, Università di Bologna 2013-2015
- winner of an **Erasmus scholarship** to study at Université Paris VII, set up by the European Union 2010-2011

REFeree FOR SCIENTIFIC JOURNALS

I have been referee for **Communications in Mathematical Physics** and **Journal of Statistical Physics**.

RESEARCH STATEMENT

Keywords: Statistical Mechanics of equilibrium, disordered systems, monomer-dimer models, deep Boltzmann machines, Langevin dynamics

I am currently working on the dynamics of n-dimensional system made up with two interacting subsystems that are coupled by a potential $V(x,y)$ but evolve in different time scales and different thermal baths. My interest in this stochastic differential equation comes from its possible connection with spin glass theory and aging phenomena. Under suitable general hypothesis on the potential, this “toy dynamics” converges to an explicit measure when the time scales are widely different. This rigorous result has been obtained together with Prof. Nicolas Macris (EPFL) and Dr. Emanuele Mingione (UniBo) [17]. We are interested in improving the control on the convergence rate to the stationary measure by means of logarithmic Sobolev inequalities for large systems. We are also investigating the violation of fluctuation-dissipation theorem for these systems. A further perspective is to study the efficiency of this two-temperatures measure -and related algorithms- in inference problems out of the Bayesian optimal setting, i.e., when the signal probability distribution is not known a priori.

A research topic I've worked on in the last years are the connections between Statistical Mechanics and Machine Learning, in particular the Deep Boltzmann Machines which are used for image recognition and reconstruction. They are systems of interacting spins (also neurons, units) arranged on a K-layers structure. Spins in the first layer codify the visible image and interact with spins in the second layer, that in turn interact with spins in the third layer, and so on. Given a training dataset of images, machine learning algorithms aim at reconstructing the network weights, i.e., the strength of interactions, in order to minimize, e.g., the KL-divergence between the empirical measure and the Gibbs measure marginalized on the visible layer. Together with Prof. Adriano Barra (UniSalento), Dr. Francesco Camilli (Unibo), Prof. Pierluigi Contucci (Unibo), Dr. Emanuele Mingione (Unibo), we have studied the statistical mechanics properties of the Deep Boltzmann Machines, assuming independent Gaussian interactions. These are disordered systems, where the Gibbs measure is a random measure. For centered interactions we have studied the replica-symmetric phase, using a bound for the quenched pressure in terms of K Sherrington-Kirkpatrick models ([12,14]). For Nishimori interactions, i.e., Gaussian interactions with equal mean and variance, we managed to completely solve the model and find a phase transition ([15]). The Nishimori identities and inequalities allowed to solve also models with elliptic interaction among spin families ([13]).

Both in Deep Boltzmann Machine with centered interactions and Nishimori interactions, it turns out that the region of parameters where the overlap between two configurations vanishes (and so there is no learning) can be shrunk by choosing a specific geometry of the layers. This observation ([14,15]) follows from the properties of Heilmann-Lieb polynomials, which are partition functions of monomer-dimer models.

It was interesting to find a link between Deep Boltzmann Machines and monomer-dimer systems, which I studied during and after my Ph.D. ([1-11]). These models describe the absorption of bi-atomic molecules on a lattice, where no superposition of atoms is allowed (hard-core interaction). The main result in this field was obtained by Heilmann and Lieb, who studied the complex zeros of the partition functions and proved the absence of phase transitions for any lattice geometry provided that the monomer activity is positive (empty sites are allowed). Nevertheless, if an attractive short-range interaction is considered beyond the hard-core one, then phase transitions may appear.

During my visit to Princeton University, I worked on a conjecture by Heilmann and Lieb concerning a monomer-dimer model on the lattice \mathbb{Z}^2 with attractive interaction among collinear dimers. The system was conjectured to be a liquid crystal with a nematic phase showing an order in the particles orientation but not in their position. The existence of a phase transition with spontaneous order in the orientation was proven by Heilmann and Lieb, while the absence of a complete order in the positions remained an open problem. In my work I used cluster expansion to verify the absence of a positional order in a modified model with the addition of a sufficiently large external field ([5]). Recently the conjecture for the original model has been proven by Jauslin and Lieb by advanced cluster expansion methods.

Together with the group of Bologna, we have studied monomer-dimer models in the mean field setting. We solved the model with hard-core interactions on locally tree-like random graphs, e.g., the sparse Erdős-Rényi random graph, using suitable correlation inequalities and the Heilmann-Lieb recursion relation ([1]). We introduced the monomer-dimer model on the complete graph with the addition of an attractive interaction and we solved it, finding a phase transition ([2,3]). Moreover we studied the law of large numbers, the central limit theorem and its breakdown at the critical point ([6,7]). We worked on the inverse problem, that is the reconstruction of dimer activities from the observations ([8]). We computed the finite volume corrections for the main thermodynamic quantities, using a suitable integral representation of the partition function ([10]).