COMBINATORIAL OPTIMIZATION **Written test of June 13th, 2019**

Given name: |__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|

Family name: |__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|

Id. number: |__|__|__|__|__|__|

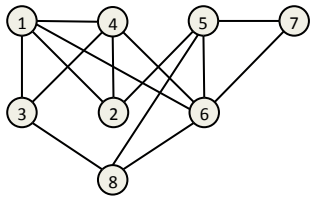
1. **Implications.** Consider the inequalities

$$4x_1 - x_2 \leq 6 \qquad 7x_1 + 5x_2 \leq 24 \qquad x_1 + 5x_2 \leq 12$$

and distinguish those implied by those not implied by the system

$$\begin{aligned} x_1 + 2x_2 &\leq 6 \\ 3x_1 - 2x_2 &\leq 6 \\ 2x_1 + 3x_2 &\leq 6 \end{aligned}$$

2. **A sparse subgraph** (*Intermediate, II part*). Given a graph $G = (V, E)$ as that shown in the figure, call $f(X)$ the number of arcs of the subgraph of G induced by X, for any $X \subseteq V$. Formulate as 01 LP the problem of finding di trovare an X of at least k elements characterized by the least possible value of $f(X)$. Show how one can formulate this problem in terms of 01 LP starting from expression of propositional calculus, rewriting them in conjunctive normal form and then obtaining a system of linear inequalities fulfilled by all the solution vectors.



3. **Independent influencers**. Imagine that the graph $G = (V, E)$ of Problem 2 describes the relations among the participants to a social network. A set $X \subseteq V$ of persons is considered as independent as the number $f(X)$ of arcs belonging to the subgraph of G induced by X, and is considered as influent on the other participants as the number $t(X)$ of arcs with one endpoint in X and the other in $V - X$. Modify the 01 LP formulation of Problem 3 by replacing the constraints on the cardinality of X with conditions that ensure a value of $t(X)$ not less than a given k. Also in this case, please show how one can obtain the formulation starting from expressions of propositional calculus.

4. **A web business, in theory** (*Intermediate, II part*). A guy has to buy something in the web, and after some search finds n sellers of the product he wants. The generic seller k offers a lot of m_k items of this product at price p_k. The buyer wishes to get as many items as he can while maintaining the average price under a prescribed value of p_0 euros. Formulate the problem as 0-1 linear programming and show how to construct an equivalent 0-1 knapsack problem from your formulation.

5. **A web business, in practice** (*Intermediate, II part*). Using the numbers in the table below and assuming $p_0 = 5$€, solve Problem 4 via branch-and-bound.

lot (k)	1	2	3	4	5	6	7
price asked (p_k)	160	145	127	110	85	70	52
amount offered (m_k)	29	27	24	21	17	15	12