DECISION MODELS, OPTIMIZATION MODELS & ALGORITHMS **Final test, Jan 13th 2020**

Given name: |__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|

Family name: |__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|

Id. number: |__|__|__|__|__|__|

1. **Clustering**. A set R of n-dimensional binary records $\mathbf{r}_1, \ldots, \mathbf{r}_m$ has to be divided into p subsets $R_1, \ldots, R_p$ according to similarities. Each R_k is formed by choosing a record of R as its centre and assigning to it some other records. Formulate the problem of finding the centres and a partition that minimizes the sum of the Hamming distances of the elements of R to the centre they are assigned to.
   Fill the matrix of Hamming distances and write the formulation for $p = 2$ in the particular case below:

$\mathbf{r}_1$	1001011
$\mathbf{r}_2$	0001010
$\mathbf{r}_3$	1001101
$\mathbf{r}_4$	0101011
$\mathbf{r}_5$	1101001

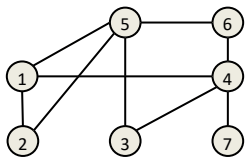
$\mathbf{r}_1$	$\mathbf{r}_2$	$\mathbf{r}_3$	$\mathbf{r}_4$	$\mathbf{r}_5$

   Remove then integrality clauses and write the dual of the problem so obtained.

2. **Projection**. Apply Fourier-Motzkin method to determine whether the polyhedron

$$x_1 + 2x_2 - x_3 \leq 2 \qquad 2x_1 - 3x_2 + x_3 \leq 6 \qquad -4x_1 + x_2 + x_3 \leq 4$$

   is empty or not, and to find one of its points in the affirmative case.

3. **Separation**. Given a set $V = \{\mathbf{v}_1, \ldots, \mathbf{v}_m\}$ of n-vectors partitioned into subsets A, B, formulate as mixed integer programming the problem of finding a linear separator that minimizes the number of outliers, that is, of elements of A and B that, after separation, lie in the "wrong" part (B or A, respectively). In the formulation, assume the zero-degree term of the separator normalized to 1.
   Suppose then that you know a symmetric binary relation defined on V — you can imagine it as an undirected graph $G = (V, E)$. Modify the problem formulated by adding to the objective function (still to be minimized) a term that counts the number of pairs formed by an outlier and a non-outlier which both lie in A or both in B and are in the given relation.

4. **Transversal**. Without reformulating it as a stable set problem, find by branch-and-bound the smallest transversal set of the graph depicted in the figure aside. Start branching on node 1 and use the combinatorial bounds provided by König's Matching Theorem.